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THE STRUCTURE OF THE PROJECTIVE INDECOMPOSABLE MODULES OF $\hat{3}M_{22}$ IN CHARACTERISTIC 2

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ABSTRACT. This paper presents the socle series of the projective indecomposable modules for the triple cover $\hat{3}M_{22}$ in characteristic 2. The results are obtained by computational means; the methods as well as the constructive approach are explained.

INTRODUCTION

In this note we want to describe how to determine the Loewy and socle structure of the projective and indecomposable modules (PIMs) for the triple cover $\hat{3}M_{22}$ of the simple Mathieu group M_{22} in characteristic 2. The knowledge of such structures is not only of interest to modular representation theory but also to the study of the cohomology of the underlying group. The computer will be the main tool to achieve this goal; in particular, R. Parker's "MEAT-AXE" package [4] and the Computer Algebra System CAYLEY [1] will be applied. We point out that among all the PIMs, the projective cover $P(1)$ of dimension 6272 is the most challenging one; to our knowledge this is the biggest module whose socle series has been determined by computational means.

The notation used is standard. In particular, $P(S)$ denotes the projective cover of the module S; moreover, whenever convenient, modules will be denoted by their dimensions. For entries in character tables we refer to [2], e.g., $bn = (-1 + i\sqrt{n})/2$ for $n \equiv 3 \bmod 4$.

1. PREREQUISITES ON IRREDUCIBLES AND PIMs

In this section we collect some data on the irreducibles and PIMs of $\hat{3}M_{22}$ in characteristic 2, which will be used later for the actual computer construction of the PIMs.

1.1. **Proposition.** *Let* $G \cong \hat{3}M_{22}$; *then the following hold*:

(1) GF(4) *is a splitting field for* G.
(2) *The* 2-*modular character table of* G *is as follows*:

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	$1A$	$3A$	$5A$	$7A$	B^{**}	$11A$	B^{**}
	1	1	1	1	1	1	1
	10	1	0	$b7$	**	-1	-1
	10	1	0	**	$b7$	-1	-1
	34	-2	-1	-1	-1	1	1
	70	-2	0	0	0	$-1+b11$	**
	70	-2	0	0	0	**	$-1+b11$
	98	-1	-2	0	0	-1	-1
	1	3	5	7	7	11	11
	3		15	21	21	33	33
	3		15	21	21	33	33
02	6	0	1	-1	-1	$-b11$	**
02	15	0	0	1	1	$-1+b11$	**
02	45	0	0	$b7$	**	1	1
02	45	0	0	**	$b7$	1	1
02	84	0	-1	0	0	$1-b11$	**
02	384	0	-1	-1	-1	-1	-1

(3) *The irreducible* $\mathrm{GF}(4)-G$ *modules fall into five blocks as follows*:

$$B_0 = \{1, 10a, 10b, 34, 70a, 70b, 98\},$$
$$B_1 = \{6a, 15a, 45a, 45c, 84a\},$$
$$B_2 = \{6b, 15b, 45b, 45d, 84b\},$$
$$B_3 = \{384a\},$$
$$B_4 = \{384b\}.$$

Here, B_0 *is the principal block containing only nonfaithful modules, whereas the remaining blocks contain only faithful modules. Moreover,* B_{i+1} *is the dual and Galois-conjugate of* B_i *for* $i \in \{1, 3\}$.

(4) *Let* D_i *and* C_i *denote the decomposition matrix and the Cartan matrix of* B_i, *respectively, for* $i \in \{0, 1\}$; *then we have*:

D_0:

	1	$10a$	$10b$	34	$70a$	$70b$	98
1	1	0	0	0	0	0	0
21	1	1	1	0	0	0	0
45	1	1	0	1	0	0	0
45	1	0	1	1	0	0	0
55	1	1	1	1	0	0	0
99	1	0	0	0	0	0	1
154	2	1	1	1	0	0	1
210	4	2	2	2	0	0	1
231	3	1	1	2	1	1	0
280	4	2	2	2	1	0	1
280	4	2	2	2	0	1	1
385	5	2	2	3	1	1	1

C_0 :

	1	10*a*	10*b*	34	70*a*	70*b*	98
1	92	42	42	50	12	12	20
10*a*	42	21	20	23	5	5	9
10*b*	42	20	21	23	5	5	9
34	50	23	23	29	7	7	10
70*a*	12	5	5	7	3	2	2
70*b*	12	5	5	7	2	3	2
98	20	9	9	10	2	2	6

D_1 :

	6*a*	15*a*	45*a*	45*c*	84*a*
21	1	1	0	0	0
45	0	0	1	0	0
45	0	0	0	1	0
99	0	1	0	0	1
105	1	1	0	0	1
105	0	1	1	1	0
210	1	2	1	1	1
231	2	3	1	1	1
231	2	3	1	1	1
330	2	4	1	1	2

C_1 :

	6*a*	15*a*	45*a*	45*c*	84*a*
6*a*	15	24	7	7	10
15*a*	24	42	13	13	18
45*a*	7	13	6	5	5
45*c*	7	13	5	6	5
84*a*	10	18	5	5	9

(5) *The Brauer characters of the PIMs are as follows*:

	1*A*	3*A*	5*A*	7*A*	*B***	11*A*	*B***
	3		15	21	21	33	33
	3		15	21	21	33	33
P(1)	6272	8	2	0	0	2	2
P(10*a*)	2816	8	1	*b*7 − 1	**	0	0
P(10*b*)	2816	8	1	**	*b*7 − 1	0	0
P(34)	3456	0	1	−2	−2	2	2
P(70*a*)	896	−4	1	0	0	*b*11	**
P(70*b*)	896	−4	1	0	0	**	*b*11
P(98)	1408	4	−2	1	1	0	0
P(6*a*)	1920	0	5	2	2	−*b*11	**
P(15*a*)	3456	0	6	5	2	2	2
P(45*a*)	1152	0	2	1 + *b*7	**	3 + *b*11	**
P(45*c*)	1152	0	2	−*b*7	**	3 + *b*11	**
P(84*a*)	1536	0	1	3	3	1 − *b*11	**
P(384*a*)	384	0	−1	−1	−1	−1	−1

Proof. The claims follow easily from [2, 5] and straightforward calculations. □

1.2. **Proposition.** *Let* $G \cong \hat{3}M_{22}$, *and let* L, A *and* F *be subgroups of* G *such that* $L \cong SL_3(4) \cong \hat{3}L_3(4)$, $A \cong \hat{3}\,\mathrm{Alt}_7$, *and* $F \cong Z_3 \times F_{55}$; *then the following hold:*

(1) $\mathrm{St}_L \uparrow^G = P(98)$, *where* St *denotes the projective irreducible Steinberg module of* L.
(2) $P(4a)_A \uparrow^G = P(98) \oplus P(10b)$ *and* $P(4b)_A \uparrow^G = P(98) \oplus P(10a)$, *where* $4a$ *and* $4b$ *denote the two different* 4*-dimensional irreducibles of* A.
(3) $6a \otimes 384b = P(98) \oplus P(70a)$ *and* $6b \otimes 384a = P(98) \oplus P(70b)$.
(4) $15a \otimes 384b = P(98) \oplus P(70b) \oplus P(34)$.
(5) $1_F \uparrow^G = P(70a) \oplus P(70b) \oplus P(1)$.
(6) $6b \otimes 384b = 2 * 384a \oplus P(84a)$.
(7) $10b \otimes 384a = 3 * 384a \oplus P(84a) \oplus P(45a)$ *and* $10a \otimes 384a = 3 * 384a \oplus P(84a) \oplus P(45c)$.
(8) $6a \otimes P(70b) = 5 * 384a \oplus P(15a)$.
(9) $\lambda_F \uparrow^G = 6 * 384a \oplus P(45a) \oplus P(45c) \oplus P(84) \oplus P(6a)$ *with a suitable nontrivial* 1*-dimensional module* λ *of* F *with* $\ker \lambda \cong F_{55}$.

Proof. Straightforward calculations using (1.1) □

1.3. *Remark.* In order to construct matrix representations of $G \cong \hat{3}M_{22}$, we start off with the following 12-dimensional representation of $G_0 \cong \hat{3}\,\mathrm{Aut}(M_{22})$ over GF(2):

$$g_8 := \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0\\ 1&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&1&1&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&1&0&0&0&0&0\\ 0&0&1&1&0&0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0\\ 0&1&1&0&1&0&0&1&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 1&0&0&1&0&1&0&0&0&1&0&0\\ 1&0&0&1&1&0&0&1&0&0&1&0\\ 1&1&1&1&0&1&0&0&0&1&0&1 \end{pmatrix}$$

$$g_{14} := \begin{pmatrix} 0&1&0&0&1&0&0&0&0&0&0&0\\ 1&1&0&0&0&0&0&0&0&0&0&0\\ 0&1&1&1&1&1&0&0&0&0&0&0\\ 1&0&1&0&1&1&0&0&0&0&0&0\\ 1&1&0&0&1&0&0&0&0&0&0&0\\ 1&1&0&1&1&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&1&0&0&1&0\\ 0&0&0&0&0&0&1&1&0&0&0&0\\ 0&0&0&0&0&0&1&1&1&1&1&1\\ 0&0&0&0&0&0&1&1&1&1&0&0\\ 0&0&0&0&0&0&0&1&0&0&1&0\\ 0&0&0&0&0&0&0&1&0&1&1&1 \end{pmatrix}$$

Then $o(g_8) = 8$, $o(g_{14}) = 14$ as well as $G_0 := \langle g_8, g_{14}\rangle \cong \hat{3}\,\mathrm{Aut}(M_{22}) \cong G : \mathbb{Z}_2$ and $G_0' = \langle g_8, (g_{14})^2\rangle \cong G$. This representation can easily be obtained using the information given in [3]. Note that if the 12-dimensional representation given above for G is read over GF(4), then it splits as $6a \oplus 6b$. So we are in a position to produce all the irreducibles (and further modules) by the standard method of taking tensors, exterior squares, etc., and chopping these into simple constituents.

2. Fingerprints

In this section we present some very elementary, but nonetheless useful, observations on a concept first introduced by R. Parker [4].

2.1. **The setting.** Let G be a finite group, F a finite field of characteristic $p > 0$, and let $I := \mathrm{Irr}(FG)$ be a complete set of nonisomorphic simple FG-modules. Given an FG-module M, then φ_M denotes the corresponding representation $G \to \mathrm{GL}(M)$. Finally, the left null-space of a matrix X is denoted by $\mathrm{NS}(X)$.

An element $w \in FG$ is called a *characteristic fingerprint* (CFP) of type J, $J \subseteq I$, provided

$$\dim(NS(\varphi_S(w)))\begin{cases} = 0 & \text{for } S \in I\backslash J, \\ > 0 & \text{for } S \in J. \end{cases}$$

For the sake of convenience, a CFP of type I is also called an *all-round fingerprint* (AFP). Clearly, base transformations do not affect the dimensions of null-spaces, and so the definition of CFPs depends only on the isomorphism type of the simple modules involved.

Note that if F is a splitting field of G, then there exists an orthogonal decomposition $FG = \bigoplus e_i FG$ with primitive idempotents e_i such that e_iFG is a projective indecomposable FG-module. Moreover, for each $S \in I$ there exists a suitable e_s among the e_i's such that $e_sFG \cong P(S)$ as well as

$$\dim(Te_s) = \begin{cases} 0 & \text{if } T \in I\backslash\{S\}, \\ 1 & \text{if } T = S. \end{cases}$$

Thus, for each $\varnothing \neq J \subseteq I$ the element $w_J := 1 - \sum_{S\in J} e_s$ is a CFP of type J having nullity 1 on each $S \in J$.

We are now in a position to list some observations whose proofs are left as easy exercises for the reader.

2.2. **Proposition.** *Let M, N be FG-modules, and let $w \in FG$; then the following hold:*

(1) *CFPs respect direct sums, i.e., $NS(\varphi_{M\oplus N}(w)) \cong NS(\varphi_M(w)) \oplus NS(\varphi_N(w))$.*

(2) *CFPs respect Galois automorphisms of F, i.e., $NS(\varphi_{M^\sigma}(w)) = NS(\varphi_M(w))^\sigma$ for $\sigma \in \mathrm{Gal}(F)$.*

(3) *Suppose that w is a CFP of type J and that $\mathscr{X}$ is the set of all FG-submodules X of M such that $X/\mathrm{rad}(X)$ is the only simple composition factor*

of X contained in J; moreover, put $V = NS(\varphi_M(w))$. Then for each $X \in \mathscr{X}$ there exists $v \in V$ such that the $\varphi_M(G)$-closure $\langle v^{\varphi_M(G)}\rangle$ equals X, and hence $\langle V^{\varphi_M(G)}\rangle \geq \langle \mathscr{X}\rangle$. Also, $\langle \mathscr{X}\rangle = \mathrm{soc}_{FG}(M)$ whenever w is an AFP.

2.3. **Example.** Here we illustrate the fact that in the context of (2.2.3) the closure $\langle NS(\varphi_M(w))^{\varphi_M(G)}\rangle$ does not necessarily contain all the simple composition factors isomorphic to elements of J which do occur in M.

Take $G = \langle a\rangle \cong \mathbb{Z}_2$ and $F = \mathrm{GF}(2)$; then FG has only one simple module, namely the trivial on 1_G. Consider $M = FG$ with $\varphi_M(a) = \left(\begin{smallmatrix}1&0\\1&1\end{smallmatrix}\right)$. So, $w = 1 + a$ is a CFP for 1_G with $\varphi_M(w) = \left(\begin{smallmatrix}0&0\\1&0\end{smallmatrix}\right)$, and $N = NS(\varphi_M(w)) = \langle(1, 0)\rangle$ is G-invariant with $N \cong M/N \cong 1_G$.

2.4. *Remark.* Note that in view of the last remark in (2.2) we have a method to find a submodule still containing $\mathrm{soc}_{FG}(M)$ which in general is considerably smaller than the module M and therefore can be used very effectively instead of M to calculate $\mathrm{soc}_{FG}(M)$.

Of course, the same idea applies if we are interested only in the J-component of the socle $(J \subseteq \mathrm{Irr}(FG))$.

3. The PIMs

We are now ready to demonstrate how to get hold of a PIM for $G = \hat{3}M_{22}$ and how to calculate its socle series. In order to produce any individual PIM, we follow the strategy already described in parts (1)–(9) of Proposition (1.2).

A simple induction process yields $P(98) = \mathrm{St}_L \uparrow^G$. In the next step a further induction gives

$$P = P(4a)_A \uparrow^G = P(98) \oplus P(10b).$$

Assuming that the socle series of $P(98)$ is known, one could try and determine that for P, and so eventually find that for $P(10b)$. Although this would have worked in this particular instance, we decided to produce $P(10b)$ as a quotient of P by successively factoring out appropriate submodules, with the effect of saving CPU time and space when computing the socle series of $P(10b)$.

We now elaborate a bit further on the factor process mentioned above. In a quick precomputation we produce a list of CFPs for each simple FG-module S, aiming in particular at those of small positive nullity; in view of the comments in (2.1), the element $1 - e_s$ would be an absolutely perfect candidate, but in general it is very hard to get hold of such idempotents in a given matrix representation of G. Now we use a CFP of type $10b$ to find the uniquely determined submodule $S_{10} \cong 10b$ of P. In the next step we use elements v of $NS(\varphi_P(w))$ for a suitable CFP of type 98 to check if $\langle v^{\varphi_P(G)}\rangle \cap S_{10} = 0$; if so, we take the quotient $P/\langle v^{\varphi_P(G)}\rangle$ and continue this way until the resulting module is $P(10b)$. Notice that by taking w of type 98, we ensure that we factor out all of $P(98)$ (see (2.2.1) and (2.2.3)). Furthermore, note that $\dim(NS(\varphi_{P(98)}(w)))$ should be comparatively small in order to minimize the number of v's to be tested.

In this way we follow Proposition (1.2) as a guideline and produce one new PIM at each time. Finally, we mention that in producing $P(6a)$, a modification of the quotient process turned out to be very useful inasmuch as instead of taking null-spaces of CFPs (their dimension being too big) we take fixed point spaces of suitable subgroups of G.

Next we proceed to describe the algorithmic approach to determine the socle series of a given PIM of G. For this, we use an enhanced version of R. Parker's "MEAT-AXE" package [4] together with the Computer Algebra System CAYLEY [1], with an implementation of various representation-theoretic algorithms of G. Schneider described in [6]. Since these algorithms can deal only with modules of a few hundred dimensions, we first of all construct a submodule still containing the socle by taking G-closures of null-spaces of suitable AFPs and fixed point spaces of Sylow p-subgroups of G. If the resulting module is still too big to be fed into a direct computation of the socle, we use fixed point spaces of suitable subgroups and CFPs of type J_i $(\subset I)$ according to a decomposition of I into disjoint subsets $J_1, \dots, J_r$ in order to calculate the J_i-components of the socle. Now restart this process by considering the appropriate quotient-module, thus calculating successively the socle layers of the given FG-module.

We close this section by giving some data concerning the performance of the algorithms described above:

X	$\dim(X)$	CPU time for calculation of the socle series of X
$P(70a)$	896	1:40 hrs
$P(98)$	1408	3:00 hrs
$P(70a) \oplus P(98)$	2304	13:20 hrs
$P(15a)$	3456	19:10 hrs
$P(1)$	6272	62:00 hrs

Also we note that the factor process to get $P(70a)$ as a quotient of the module $6a \otimes 384b \cong P(98) \oplus P(70a)$ only took $2\frac{1}{4}$ hours of CPU time, which clearly shows the advantage of this approach compared with the direct computation of the socle series of $6a \otimes 384b$. Finally, we mention that all these calculations were carried out on an IBM RS 6000 (model 320 H).

Having explained all the ingredients of the algorithms to compute the socle series, we are now ready to state the results; but before doing so, we make the following obvious

Remark. The socle series and Loewy series of all the nonsimple PIMs of $\hat{3}M_{22}$ in characteristic 2 can be obtained from those whose socle is contained in the principal block B_0 or in the block B_1, simply by an application of duality or Galois-conjugation.

Here is our

Main Theorem. *The socle series of the PIMs,* $P(S)$, $P(S)$, $S \in B_0 \cup B_1$, *for the group* $\hat{3}M_{22}$ *in characteristic* 2 *are as follows*:

P(1)

1
10b 34
1 10a 34 70a 70b
1 10a 10b 98
1 1 10a 10a 34 34 70a 70b
1 1 1 1 10a 10b 34 34
1 1 1 10a 10b 34 34 98
1 1 1 1 1 10a 10b 10b 34 34 34 98 98
1 1 1 1 1 10a 10b 10b 10b 34 34 70a 70b 98
1 1 1 1 10a 10a 10a 10b 10b 10b 10b 34 34 70a 70b 98
1 1 1 1 10a 10a 10a 10b 10b 10b 34 34 34 70a 70b
1 1 1 1 10a 10a 10a 10b 34 34 34 70a 70b 98
1 1 1 1 1 1 10a 10a 10a 10a 10a 10b 34 34 98 98
1 1 1 1 1 1 1 1 1 1 10b 10b 10b 34 34 34 34 98
1 1 1 10a 10a 10a 10b 10b 10b 10b 34 34 34 34 34 70a 70b 98 98 98
1 1 1 1 1 1 1 1 1 10a 10a 10a 10a 10b 10b 10b 34 70a 70b
1 1 1 1 1 1 10a 10a 10b 10b 10b 10b 10b 34 34 34 34 70a 70b
1 1 1 1 10a 10a 10a 10a 10a 10b 34 34 34 34 70a 70b 98 98 98
1 1 1 1 1 1 1 1 10a 10a 10b 10b 34 34
1 1 1 1 1 10a 10b 10b 10b 34 34 34 98 98
1 1 1 10a 10a 10b 34 34 70a 70b 98
1 1 1 10b 10b 34
10a 34 70a 70b 98
1

P(10a)

10a
1
34
1
10b 98
1 10a 10b 34 70a 70b
1 10a 10a 10b 34
1 1 10a 34
1 1 1 10a 34 34 98
1 1 1 1 10b 34 98
1 1 1 10b 10b 10b 34 98
1 10a 10b 10b 10b 34 34 70a 70b
1 10a 10a 10a 10b 34 70a 70b
1 1 1 10a 10a 10a 34
1 1 1 1 34 34 34 98
1 1 1 10a 10a 10b 34 98 98
1 1 1 1 1 10b 10b 34
1 10a 10b 10b 10b 34 34 70a 70b
1 1 10a 10a 10a 34 70a 70b
1 1 1 10b 34
1 10a 34 98 98
1 1 10b
10b 34
10a

P(10b)

10b
10a 34
1 10a
1 34
1 1 34
1 10b 98
1 10b 10b 34 98
1 10a 10b 34 70a 70b
1 10a 10a 10b 34 70a 70b
1 1 1 10a 10a 34
1 1 1 34 34 98
1 1 1 10a 10a 10b 34 98
1 1 1 1 1 10b 10b 10b 34
10a 10b 10b 10b 34 34 34 70a 70b 98
1 1 1 10a 10a 10a 10a 70a 70b
1 1 1 1 10b 10b 34 34
1 1 10a 10a 34 34 98 98
1 1 1 1 1 10a 10b 34
1 1 10b 10b 10b 34 34
1 10a 10a 34 70a 70b 98
1 1 10b
34 98
1 10a
10b

P(34)

34
1 10a
1 10b
10a 34 34 70a 70b
1 1 10a 98
1 1 10b 34
1 1 10a 10b 34 34 70a 70b
1 1 1 10a 10b 98
1 1 10a 10b 10b 34 34
1 1 10a 10b 34 34 70a 70b 98
1 1 1 10a 10a 10b 98
1 1 1 10a 10b 10b 34 34
1 1 10a 10b 34 34 34 70a 70b 98
1 1 1 1 10a 10a 10a 10b 98
1 1 1 1 1 10b 10b 10b 34 34
1 10a 10a 10b 34 34 34 70a 70b 98
1 1 1 1 10a 10a 10b 34 34
1 1 1 1 10a 10b 10b 34 34
1 1 10a 10a 10b 34 70a 70b 98 98
1 1 1 10a 10b 34
1 1 10b 34 34
1 10a 70a 70b 98
1 10b
34

P(70a)

70a
1
34
1
10b
34
10a 70b
1 10a
1 34
1 98
1 10b
10b 34
10a 70a
1 10a
1 34
1 98
1 10b
10b 34
10a 70b
1
34
1
70a

P(70b)

70b
1
34
1
10b
34
10a 70a
1 10a
1 34
1 98
1 10b
10b 34
10a 70b
1 10a
1 34
1 98
1 10b
10b 34
10a 70a
1
34
1
70b

P(98)

98
1
10b 34
10a
1 10a
1 1 34
1 10b 98
1 10b 34
10a 10b 34 70a 70b
1 10a
1 1 34
1 10a 34 98
1 1 1 10b
10b 10b 34
10a 10a 70a 70b 98
1 1 1
34 34 98
1 1 10a
1 10b 10b
10a 34
1
98

P(6a)

6a
15a 15a
6a 6a 84a
15a 15a 15a
6a 6a 45a 45c 84a 84a 84a
15a 15a 15a 15a 15a 45c
6a 6a 6a 15a 15a 45a 45a 45c 84a 84a
6a 15a 15a 15a 15a 45a 45c
6a 6a 6a 15a 45a 45c 45c 84a 84a
15a 15a 15a 15a 15a 45a
6a 6a 45a 45c 84a 84a
15a 15a
6a

P(15a)

15a
6a 6a 84a 84a
15a 15a 15a 15a 45a
6a 6a 6a 45c 45c 84a 84a
15a 15a 15a 15a 15a 15a 15a 15a 15a 45a
6a 6a 6a 6a 6a 45a 45a 45a 45c 45c 84a 84a 84a 84a 84a
6a 6a 15a 15a 15a 15a 15a 15a 15a 15a 45c 45c 45c
6a 6a 6a 6a 15a 15a 15a 15a 15a 45a 45a 45c 84a 84a 84a
6a 15a 15a 15a 15a 15a 15a 45a 45a 45a 45c 84a
6a 6a 6a 6a 6a 15a 45a 45c 45c 45c 84a 84a 84a
15a 15a 15a 15a 15a 15a 15a 45a
6a 6a 45a 45c 84a 84a
15a

P(45a)

45a
45c
15a 15a
6a 45a 84a 84a
6a 15a 15a 45c
6a 15a 15a 15a 45a
6a 15a 45a 45c 84a
6a 6a 15a 45c 84a
15a 15a 15a 45a
6a 45c 84a
15a
45a

P(45c)

45c
15a
45a 84a
6a 15a 45c
15a 15a 15a 45a
6a 6a 45a 45c 84a 84a
6a 15a 15a 45c
6a 15a 15a 15a 45a
6a 15a 45c 84a
6a 15a 84a
15a 45a
45c

P(84a)

84a
15a 15a
6a 84a
15a 15a 45a
6a 6a 6a 45c 45c 84a
15a 15a 15a 15a 15a
6a 6a 45a 45a 84a 84a
15a 15a 15a 45c 84a
6a 6a 15a 45c 84a
15a 15a 15a 45a
6a 6a 45a 45c 84a
15a 15a
84a

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